Stars of the Big Dipper: A 3-D Vector Activity

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Most teachers of introductory physics will agree that many students have difficulty with vectors, so much so that we frequently spend a week at the beginning of the semester presenting material that students should know from previous mathematics courses. This review is often quite abstract, with little or no connection to familiar contexts, and seldom includes any motivation for students to "see it again." In this paper we present a vector activity that attempts to address both these issues using the stars of the Big Dipper, in the constellation Ursa Major, as a memorable context.

The Classroom

At North Carolina State University, a section of the introductory calculus-based physics course is taught in the SCALE-UP environment (Student-Centered Activities for Large Enrollment Undergraduate Programs)[2] and uses the Matter and Interactions[3] curriculum. The classroom accommodates 99 students at 11 round tables. Each table of nine students is further broken into three groups of three students working cooperatively[4] to solve problems and carry out activities. The classroom is computer-rich with multiple laptops at each table.

Fig. 1. Photo of the stars of the Big Dipper.[1]

The Activity

The Matter and Interactions curriculum treats vectors as a single triplet, <*x*, *y*, *z*>, designating a single position in space with respect to a reference point. This triplet notation is also found in widely used calculus textbooks. Students easily work in three dimensions at once, rather than using extensive trigonometry to extract each component separately. This treatment of vectors is continued throughout the entire course, so it is important that students become proficient early in the semester. To give students the opportunity to practice using triplets, we have developed an activity using data on the stars of the Big Dipper (see Fig. 1). Close inspection reveals that what we have labeled as Mizar is actually a double star. Viewed from Earth, the two stars are aligned so they look close together, but they do not form a true binary star system. We did not include data on Alcor, Mizar's "companion" star, in our students' activity. You may want to include it for your students. It is interesting to note that from the earliest days of sky watching, Arabs and Native Americans have utilized this pair of stars as a test for visual acuity.[5]

The astronomical data shown in Table I can be

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Table I. Data on the stars of the Big Dipper. The distance from Earth, *D*, is given in light-years,[6] the right ascension, *RA*, is given in hours, and the declination, *Dec*, is given in degrees.[7]

Label	Name	*D* (ly)	*RA* (h)	*Dec* (deg)	Vector Triplet (ly)
α	Dubhe	105	11:03:43.7	61:45:03	<12.1, 92.5, -48.2>
β	Merak	80	11:01:50.5	56:22:57	<11.1, 66.6, -42.9>
γ	Phecda	90	11:53:49.8	53:41:41	<1.4, 72.5, -53.3>
δ	Megrez	65	12:15:25.6	57:01:57	<-2.4, 54.5, -35.3>
ε	Alioth	70	12:54:01.7	55:57:35	<-9.1, 58.0, -38.1>
ζ	Mizar	88	13:23:55.5	54:55:31	<-18.1, 72.0, -47.2>
80	Alcor	89	13:25:13.5	54:59:17	<-18.5, 72.9, -47.6>
η	Alkaid	210	13:47:32.4	49:18:48	<-61.8, 159.2, -122.2>

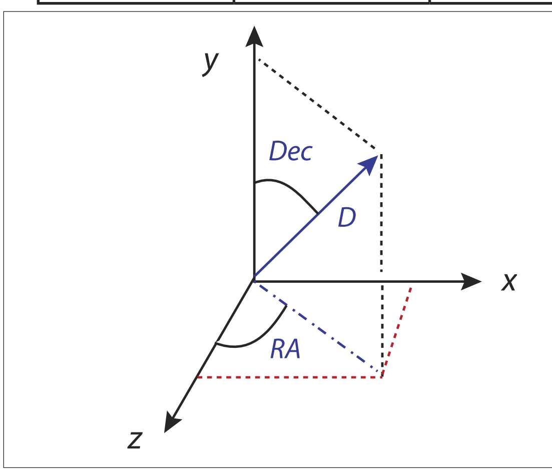

Fig. 2. Spherical coordinate projections onto Cartesian coordinates.

```
from visual import *

#look through Earth (@ origin) towards constellation
scene.forward = vector (0, 0.8, -0.5)
#zoom in so you can see it
scene.range = 1

sphere(pos = vector(12.2, 92.5, -48.2))
sphere(pos = vector(11.2,66.6, -42.8))
sphere(pos = vector(1.5, 72.5, -53.3))
sphere(pos = vector(-2.3,54.5, -35.3))
sphere(pos = vector(-9.1,58.0, -38.1))
sphere(pos = vector(-18.0,72.0, -47.2))
sphere(pos = vector(-61.6,159.3, -122.2))
```

Fig. 3. Simple VPython code for plotting the positions designated by the vectors.

readily converted to Cartesian coordinates (see Fig. 2) using $x = D \sin(Dec) \sin(RA)$, $y = D \cos(Dec)$, and $z = D \sin(Dec) \cos(RA)$. The last column of Table I shows the conversion results for each star as a vector triplet. With Earth at the origin of the coordinate system, each triplet represents the vector pointing to a star's location in space. Since teaching coordinate transformation was not the objective, this part of the activity was done in advance by the instructor via spreadsheet, and the resulting triplets were given to the students. One can imagine a suitable situation where students are given the opportunity to calculate these values on their own.

The Program

Students were asked to plot these seven locations in space as their first introduction to VPython,[8] a 3-D visual environment. The program code is shown in Fig. 3. Notice there are no output statements. Simply creating an object causes it to be displayed automatically in a three-dimensional scene. The "scene. forward" line of code tilts the viewing window to allow the student to look through the Earth (located at the origin) toward the Big Dipper. The "scene. range" command zooms the camera so the viewer is close enough to see the stars. These two commands, along with the first line, which imports the VPython package and the syntax for creating a sphere object were given to the students. Students write the code for plotting the stars using the vector triplets provided by the instructor. This allows the students to recognize that each vector represents a particular position on a coordinate system and to see the positions of each of the seven stars. They are not told in advance what they are plotting and are usually pleased to see the familiar dipper shape (see Fig. 4).

A complementary activity is to trace outlines between stars to better reproduce the shape of the Big

Dipper, i.e., "connect the dots." This is an exercise in vector subtraction. Students must recognize that the vectors from the previous part of the activity have their "tails" at the origin. So to trace the vector from one star to the next, the *relative position vector*, one must do something different with the position vectors. This gives rise to the opportunity to discuss vector addition and subtraction. Fig. 5 shows how vector addition and subtraction can be discussed within the context of this activity.

If star α has position vector **A** while star β has position vector **B,** then the relative position vector from star α to star β is given by **C** = **B** –**A.**

Since there are seven stars in the Big Dipper, this activity gives students several opportunities to practice this calculation. For example, to trace the relative position vector from Dubhe to Merak, the tail must be given by the position vector of Dubhe. The direction that this relative position vector points is therefore the difference between the position vectors of Merak and Dubhe. This, in VPython code, is:

```
DubheToMerak = cylinder (pos = Dubhe.pos, axis = Merak.pos - Dubhe.pos, ...)
```

Creates a long, thin cylinder used as a line to "connect the dots"

Position of one end of the cylinder

Axis points from one end of the cylinder to the other, the two stars' relative position vector

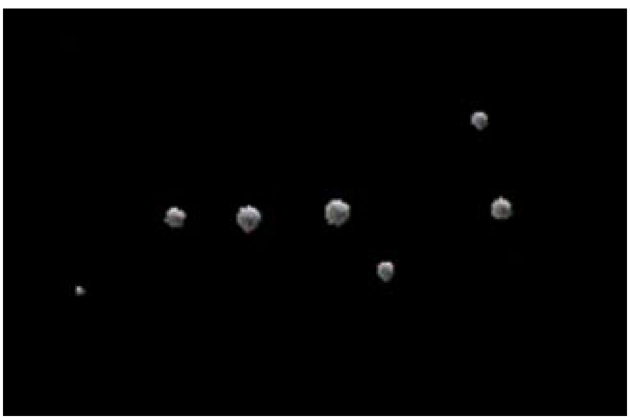

Fig. 4. VPython output from the simple code.

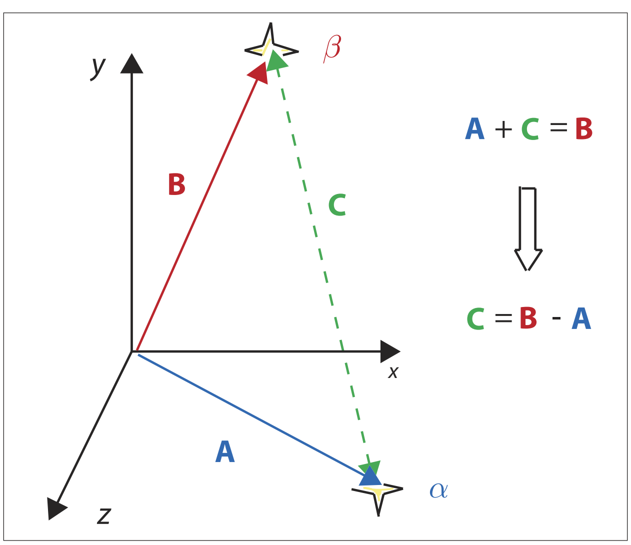

Fig. 5. Adding and subtracting vectors.

Students are given an additional "boilerplate" code to create a set of coordinate axes. Running the completed program (shown in Fig. 6) creates the viewing window shown in Fig. 7. As mentioned earlier, the viewing window is tilted such that the line of sight goes through the Earth at the origin. At this point the students can clearly see that they have reproduced the shape of the Big Dipper using only what they know about vectors.

In the 3-D visual environment, students can easily use the mouse to zoom and rotate the viewing field. With the Big Dipper, rotation in any direction will result in a completely different configuration (see Fig. 8). This allows students to see that the Big Dipper only looks the way it does from our perspective on Earth, and offers a concrete example of the idea of "point-of-view." The activity also provides reinforcement of the idea that vectors are directional and establishes the value of a "firmly positioned" origin of coordinates.

The activity continues by having students determine, on paper, the time it takes light from each of the stars to reach Earth. This provides practice finding the magnitude of the position vectors and gives students experience using different units. Hopefully, this activity gives students a memorable context in calculating magnitude of vectors as well. It takes anywhere from 65 to 210 years, depending on the star. Asking for the time it takes Alkaid's light to reach Dubhe lets students find the magnitude of a relative position vector.

```
from visual import  *

# right button mouse drag spins (Mac: option key + mouse)
# double button mouse drag zooms (Mac: command key + mouse)

################  This stuff makes labeled coordinate axes #################

d = 25    #adjust length of axes as needed
r = 0.1   #adjust thickness of axes as needed

scene.background = color.white   #this makes it easier to see when projected
scene.x = scene.y = 0
scene.width = scene.height = 800
scene.range = 1.2*d

xaxis = cylinder(pos=vector(0,0,0), axis=vector(d,0,0), radius=r)
yaxis = cylinder(pos=vector(0,0,0), axis=vector(0,d,0), radius=r)
zaxis = cylinder(pos=vector(0,0,0), axis=vector(0,0,d), radius=r)

label (pos=xaxis.pos + xaxis.axis, text='x', box=0)
label (pos=yaxis.pos + yaxis.axis, text='y', box=0)
label (pos=zaxis.pos + zaxis.axis, text='z', box=0)

############################################################################

#look through Earth (@ origin) towards constellation
scene.forward = vector(0, 0.8, -0.5)

# Plot the stars
# Earth @ (0.0, 0.0, 0.0), (x, y, z) = light yrs from Earth, radius = arbitrary
size

Dubhe=sphere(pos=vector(12.2, 92.5, -48.2), radius=1, color=color.yellow)
Merak=sphere(pos=vector(11.2,66.6, -42.8),  radius=1, color=color.yellow)
Phecda=sphere(pos=vector(1.5, 72.5, -53.3), radius=1, color=color.yellow)
Megrez=sphere(pos=vector(-2.3,54.5, -35.3), radius=1, color=color.yellow)
Alioth=sphere(pos=vector(-9.1,58.0, -38.1), radius=1, color=color.yellow)
Mizar=sphere(pos=vector(-18.0,72.0, -47.2), radius=1, color=color.yellow)
Alkaid=sphere(pos=vector(-61.6,159.3, -122.2), radius=1, color=color.yellow)

# "Connect the dots"
DubheToMerak=cylinder(pos=Dubhe.pos, axis=Merak.pos-Dubhe.pos, radius=0.2,
    color=color.blue)
MerakToPhecda=cylinder(pos=Merak.pos, axis=Phecda.pos-Merak.pos, radius=0.2,
    color=color.blue)
PhecdaToMegrez=cylinder(pos=Phecda.pos, axis=Megrez.pos-Phecda.pos, radius=0.2,
    color=color.blue)
MegrezToAlioth=cylinder(pos=Megrez.pos, axis=Alioth.pos-Megrez.pos, radius=0.2,
    color=color.blue)
AliothToMizar=cylinder(pos=Alioth.pos, axis=Mizar.pos-Alioth.pos, radius=0.2,
    color=color.blue)
MizarToAlkaid=cylinder(pos=Mizar.pos, axis=Alkaid.pos-Mizar.pos, radius=0.2,
    color=color.blue)
```

Fig. 6. VPython code for plotting the stars of the Big Dipper.

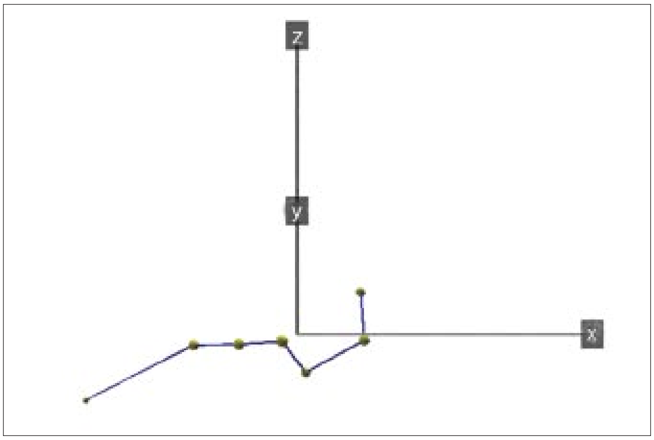

Fig. 7. VPython output of the stars of the Big Dipper.

$$\Delta\,\mathbf{r} = \mathbf{r}_{\text{Dubhe}} - \mathbf{r}_{\text{Alkaid}}$$

$$\Delta\,\text{r} = \langle 12.1 - (\text{-}61.8),\, 92.5 - 159.2,\, \text{-}\,48.2 - (\text{-}122.2)\rangle\ \text{ly}$$

$$\Delta\,\mathbf{r} = \langle 73.9,\, \text{-}\,66.7,\, 74.0\rangle\ \text{ly}$$

$$|\Delta\mathbf{r}| = \sqrt{(73.9)^2 + (-66.7)^2 + (74.0)^2} = 124\ \text{ly}.$$

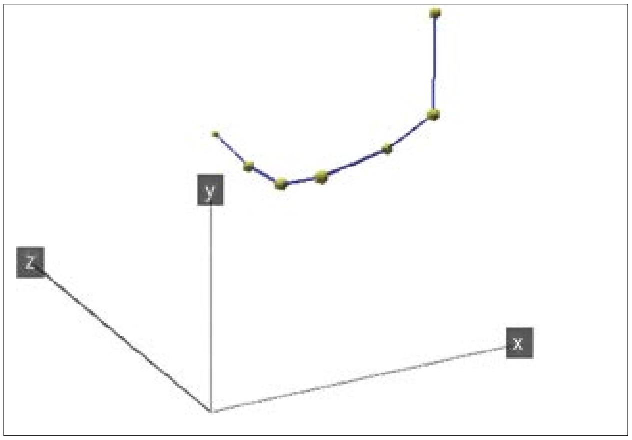

Fig. 8. Stars of the Big Dipper viewed from a rotated viewing angle.

Now we set up a scenario straight out of a science fiction story: A spaceship has been sent from Alkaid to Dubhe. The ship and descendants of the original crew are scheduled to arrive 372 years after the launch. The students happen to be their generation's navigators during the 100th anniversary of launch. Since they are steering the ship, they have to determine the appropriate velocity vector and the speed (which are clearly two different things). Assuming constant speed over the 372-year travel:

$$\mathbf{r}_\text{f} = \mathbf{r}_\text{i} + \mathbf{v}(t_\text{f} - t_\text{i}) = \mathbf{r}_\text{i} + \mathbf{v}\Delta t$$

$$\mathbf{v} = \frac{\mathbf{r}_\text{f} - \mathbf{r}_\text{i}}{\Delta t} = \frac{\langle 73.9, -66.7, 74.0\rangle \text{ly}}{372 \text{ years}}$$

$$= \langle 0.199, -0.179, 0.199\rangle \frac{\text{ly}}{\text{year}}$$

$$|\mathbf{v}| = 0.333\frac{\text{ly}}{\text{year}} = \frac{c}{3} = 1\times 10^8\ \frac{\text{m}}{\text{s}}.$$

Of course, they need to know the direction of this vector, so they must calculate the unit vector pointing from Alkaid to Dubhe. They gain insight when they realize they can calculate the unit vector of either the velocity vector or the relative position vector. Since this is purely direction, the distance units have to cancel out.

$$\Delta\mathbf{r} = \frac{\Delta\mathbf{r}}{|\Delta\mathbf{r}|} = \frac{\langle 73.9, -66.7, 74.0\rangle \text{ly}}{124\ \text{ly}} = \langle 0.596, 0.538, 0.597\rangle.$$

Finally, being good navigators, they have to find their actual position in three-dimensional space. This is easily determined since they have been traveling with a constant velocity:

$$\mathbf{r}_\mathrm{f} = \mathbf{r}_\mathrm{i} + \mathbf{v}\Delta t = \langle -61.8, 159.2, -122.2 \rangle \,\mathrm{ly}$$
$$+ \langle 0.199, -0.179, 01.99 \rangle \frac{1\mathrm{y}}{\mathrm{year}} (100 \text{ years})$$
$$\mathbf{r}_\mathrm{f} = \langle -61.8, 159.2, -122.2 \rangle \,\mathrm{ly} + \langle 19.9, -17.9, 19.9 \rangle \,\mathrm{ly}$$
$$\mathbf{r}_\mathrm{f} = \langle -41.9, 141.3, -102.3 \rangle \,\mathrm{ly}.$$

We believe that the combination of providing powerful computational visualization tools, giving students an entertaining and memorable situation to motivate their study of 3-D vectors, and having students support each other in teams results in students having a more sophisticated concept of vectors, as well as being more adept at working in three dimensions. This is evident from the relative ease that students have in dealing with unit vectors, a topic normally considered to be quite advanced.

Concluding Remarks

Students often view vectors as nothing more than just some special mathematical notation with little or no connection to their reality. The activity presented here attempts to address this issue by giving students the opportunity to see vectors "in action." The Big Dipper not only provides a familiar and memorable context within which students can practice using vectors, it also provides a direct connection between the mathematics and the physical world.

References

1. Photo obtained from http://www.digitalastro.com/constellations.html with the permission of the website author.
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6. Information obtained from http://www.m-crossroads.org/splanet/prairieskies.html.
7. Information obtained from http://www.astro.wisc.edu/~dolan/constellations/constellations/Ursa_Major.html.
8. VPython is a free and open-source programming language that offers real-time 3-D output. Available at http://vpython.org.

PACS code: 01.50.My, 95.85.-e

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Robert Beichner *is a professor of physics at North Carolina State University, where his research focuses on increasing our understanding of student learning and the improvement of physics education. Working from a base of National Science Foundation and computer industry support, he developed the popular "video-based lab" approach for introductory physics laboratories. In a separate project, he and his students are writing a series of tests, such as the TUG-K, aimed at diagnosing students' misconceptions about a variety of introductory physics topics. His biggest current project is the creation and study of a teaching/learning environment called SCALE-UP: Student-Centered Activities for Large Enrollment Undergraduate Programs. The SCALE-UP project is part of Beichner's efforts to reform physics instruction at a national level. He is the director of the PER-CENTRAL project.*

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