Linear Inverse Problems:MATLAB exercises

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Exercise #1. If $X = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ compute X^{-1} by hand.

Exercise #2. If $X = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ compute X^{-1} by using the MATLAB command `inv()`

Exercise #3. In MATLAB type in

$$X = \begin{bmatrix} 5 & 1 & 3 \\ 3 & 1 & 25 \\ -1 & 5 & 2 \\ 0 & 0 & 2 \\ -1 & -1 & -2 \end{bmatrix}$$

- Compute the transpose by typing X'.
- What are the dimensions of X? Use the command `size`(X)
- Create another rectangular matrix X. For instance, use $X =$`rand`$(300, 5)$. Is this matrix X nonsingular? Try `inv`(X)???
- Is the matrix $X^T X$ rectangular?? Type in $X' * X$. Type in `size`(X'*X). Does $(X^T X)^{-1}$ exist? Try `inv`(X'*X)??

Exercise #4. In MATLAB type in `xx`= $\begin{bmatrix} 0.8350 & 0.6670 \\ 0.3330 & 0.2660 \end{bmatrix}$

Using `v` = $\begin{bmatrix} 0.168; & 0.067 \end{bmatrix}$, find `betahat=inv`(xx)*`v`.

Using `vtilde` = $\begin{bmatrix} 0.168; & 0.066 \end{bmatrix}$, find `betatilde=inv`(xx)*`vtilde`.

Compare `betahat` and `betatilde`.

Exercise #5. In MATLAB the condition number is computed by using the command `cond`.

Using the matrix `xx` from exercise # 4, compute `cond(xx,1)`

Is the matrix `xx` well-conditioned? or is it ill-conditioned?

Exercise #6. Using MATLAB find `cond([18 6; 6 2],1)` and `inv([18 6; 6 2])`.

Exercise #7.

- In this exercise you will use the MATLAB commands `load`, `inv`,`cond`, and `plot`.
- Please make sure the files `lin_inv_prb_data1.mat`, `lin_inv_prb_data2.mat`, `lin_inv3.m`, and `lin_inv_ex7.m` are in your current working directory.
- Both files with extension `.mat` contain data stored in several variables. You may look at the names of these stored variables by using the command `who`.
- Type in the terminal `clear all`.
- Type in the terminal `load lin_inv_prb_data1.mat`
- Type in the terminal `who`, this command will display the variables stored in the file `lin_inv_prb_data1.mat`.
- Now type in again `clear all`.
- Each data file contain a matrix X, three sets of observations y, v, and w, as well as the true vector of parameters β_0 used to generate the observations. The observations stored in y, v, and w differ only by gradual increments in noise or measurement error.
- Open the m-file called `lin_inv_ex7.m`.
- For each data file, you will compute the condition number of the matrix $X^T X$, find the estimates corresponding to each set of observations, and the plot the true parameters versus the estimates.
- The estimates are found by computing,
  $\hat{\beta}_y = (X^T X)^{-1} X^T y$,
  $\hat{\beta}_v = (X^T X)^{-1} X^T v$,
  $\hat{\beta}_w = (X^T X)^{-1} X^T w$
- Is the matrix X rectangular? Is the matrix $X^T X$ ill-conditioned?
- When comparing the true vector of parameter β_0 versus the estimates $\hat{\beta}_y$, $\hat{\beta}_v$, $\hat{\beta}_w$, how can we tell which are good, reasonable or bad estimates?