

Solving the Inverse Problem for the Vibrating Beam

Dr. William LeFew

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Forming the Inverse Problem

Experimental Data:

- Displacement of the Vibrating Beam
- Times Correlating to the Displacements

Mathematical Model:

$$\ddot{y} + C\dot{y} + Ky = 0$$

$$y(0) = y_0, \dot{y}_0 = 0$$

Denote collected data as y_i and modeled data as $y(t_i; C, K)$.

Solving the Inverse Problem I

A 'Good' Solution:

- Choose a cost function.
- Minimize the cost function.

Our Cost Function:

$$L(C, K) = \frac{1}{2} \sum_{i=1}^N (y(t_i; C, K) - y_i)^2$$

Solving the Inverse Problem II

Matlab Approach:

- Load Data.
- Guess an initial C_0, K_0 . Recall that the analytical solution of the ODE is $y(t) = Ae^{-\frac{1}{2}Ct} \sin(\omega t + \phi)$, $\omega = \frac{\sqrt{4K-C^2}}{2}$. What does large or small C, K imply?
- Write a function to evaluate the cost of a given C, K .

$$L(C, K) = \frac{1}{2} \sum_{i=1}^N (y(t_i; C, K) - y_i)^2$$

- Write a function to evaluate your ode. Recall that:

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K & -C \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

This is like $\dot{z} = Az$ which gives us access to many numerical solvers.

Solving the Inverse Problem III

Matlab Approach Continued:

- Use `fminsearch` over the cost function to find optimal C , K .
- Simulate the ode for your optimal C , K .
- Plot a comparison between your data and your results.

Cautions & Tips:

- If your model starts at a maximum, arrange it such that your data does as well.
- If you cannot change where you start your data, you must change where you start your model. (what other things might you want to check to make certain your data follows the assumptions of your model?)
- Use Matlab's help command (ex. `help ode23`) to learn more about the functions we use.