Modeling the movement of a vibrating beam

Erickson Smith
Evelyn Robles Vega
Kate Costanzo

Purpose

- To attempt to model the oscillating movement of a vibrating beam
- Use characteristic equation of a spring to describe the motion of the beam
  - $$m\ddot{y} + c\dot{y} + ky = 0$$
  - has the general solution $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$
    - our system is underdamped, so y has complex roots:

$$r_1 = -(\frac{c}{2m}) + i\sqrt{\omega},$$

$$r_2 = -(\frac{c}{2m}) - i\sqrt{\omega}$$

Our formula

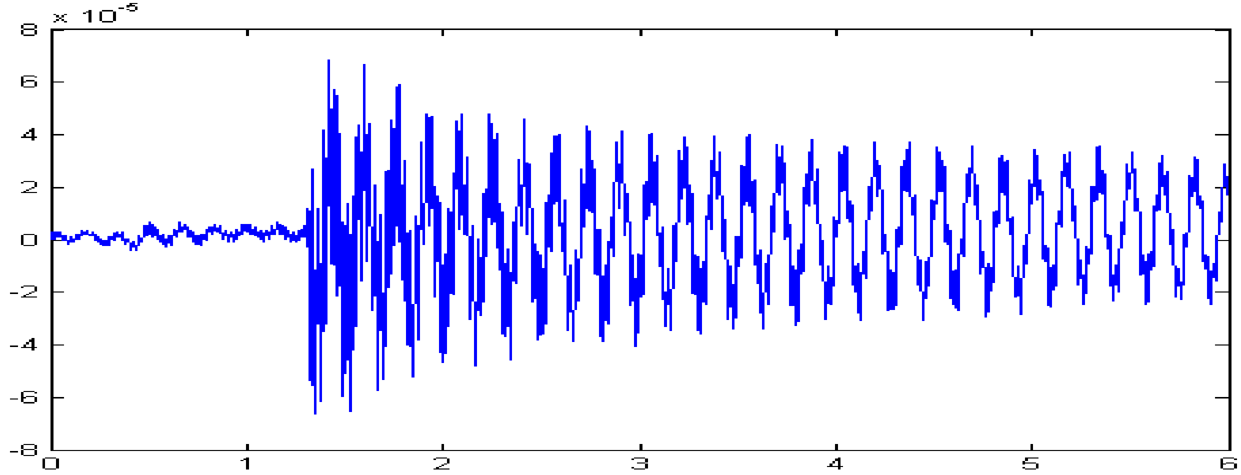
- Using Euler's formula, $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ we get a model in the form:

$$y(t) = Ae^{-\frac{ct}{2m}}\sin(\omega t + \phi)$$

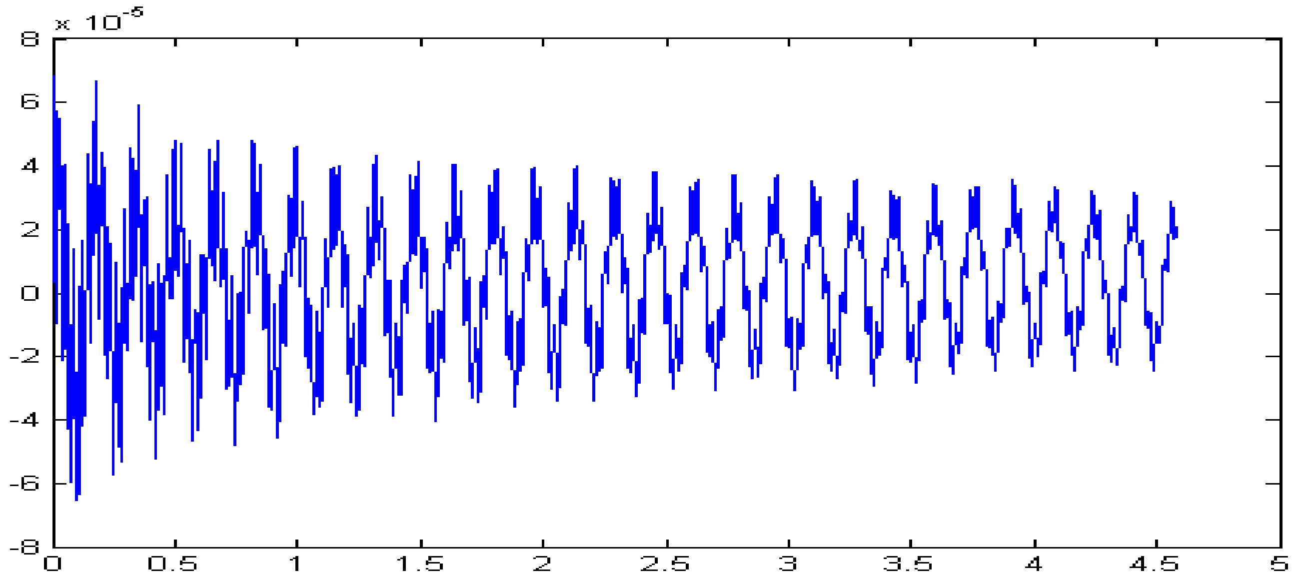
  - A = constant
  - c = coefficient of friction
  - m = mass
  - ω = √(k/m), where k = spring constant
  - ϕ = a phase shift of sine
- Eliminating the phase shift ϕ, we can replace sine with cosine to get

$$y(t) = Ae^{-\frac{ct}{2m}}\cos(\omega t)$$

Our data



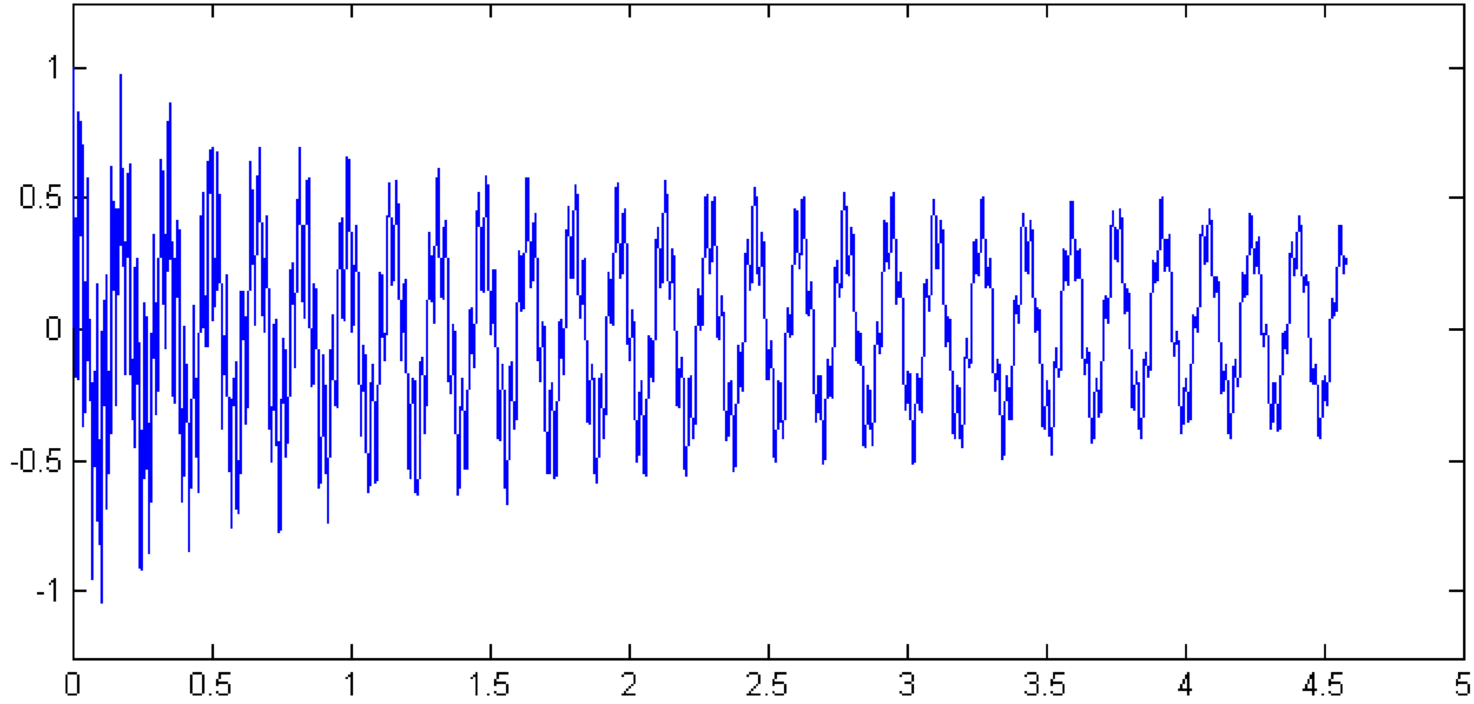
- In order to use cosine in our model, we had to cut off the data before the global maximum



Standardization

- After truncating the data, we standardized the displacement values by subtracting the mean
  (mean(y)= 2.9242×10^{-6})
- This became a new data set that was then divided by its maximum displacement
  (max(y) = 6.5695×10^{-5})
- This process gave us a standardized data set with a global maximum of 1

Standardized values



Frequency and Period

- One period is the distance between two local maxima
- Counting the local maxima, we found the displacement had 28 oscillations
- Dividing the total time ($t = 4.558$) by the number of periods, we get the average period, $T = 4.558/28$
- This is then used to find ω, which is $2\pi/T$ or $2\pi*(28/4.558)$

The exponential component

- Our model then becomes
  $$y(t) = Ae^{-(c/2m)*t}\cos(2\pi*(28/4.558)*t)$$
- The next step is to determine the exponential component of the model
- We found the coordinates for the local maxima after taking the absolute value of the standardized data
  - This gave us 57 points

Regression

- Using the 57 coordinates from the local maxima, we found the line of best fit for this data using exponential regression

Regression model

General model Exp1:

f(x) = a*exp(b*x)

Coefficients (with 95% confidence bounds):

a = 0.832 (0.7851, 0.8788)

b = -0.21 (-0.2381, -0.1818)

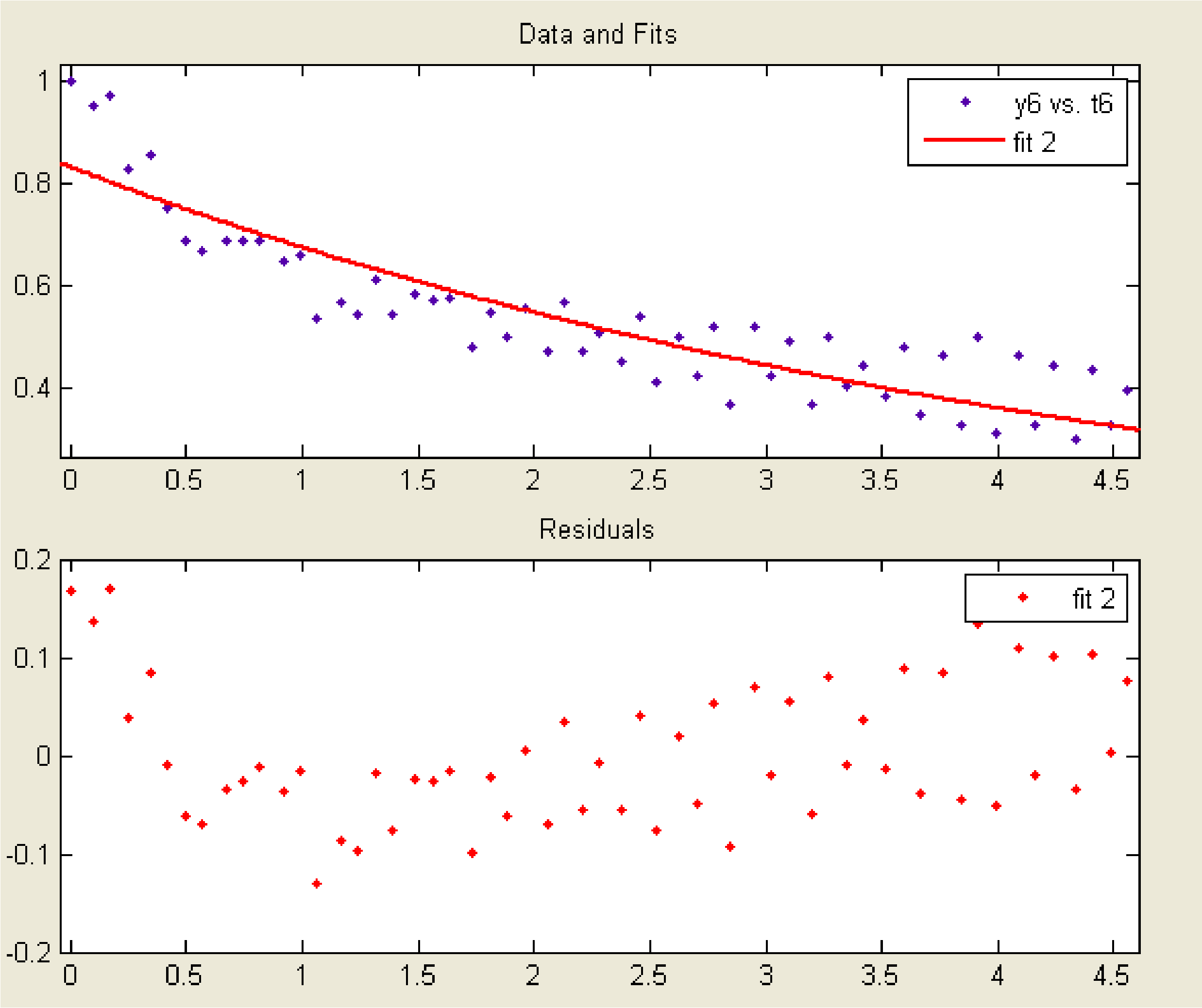
Goodness of fit:

SSE: 0.2859

R-square: 0.8053

Adjusted R-square: 0.8018

RMSE: 0.0721



Putting it all together

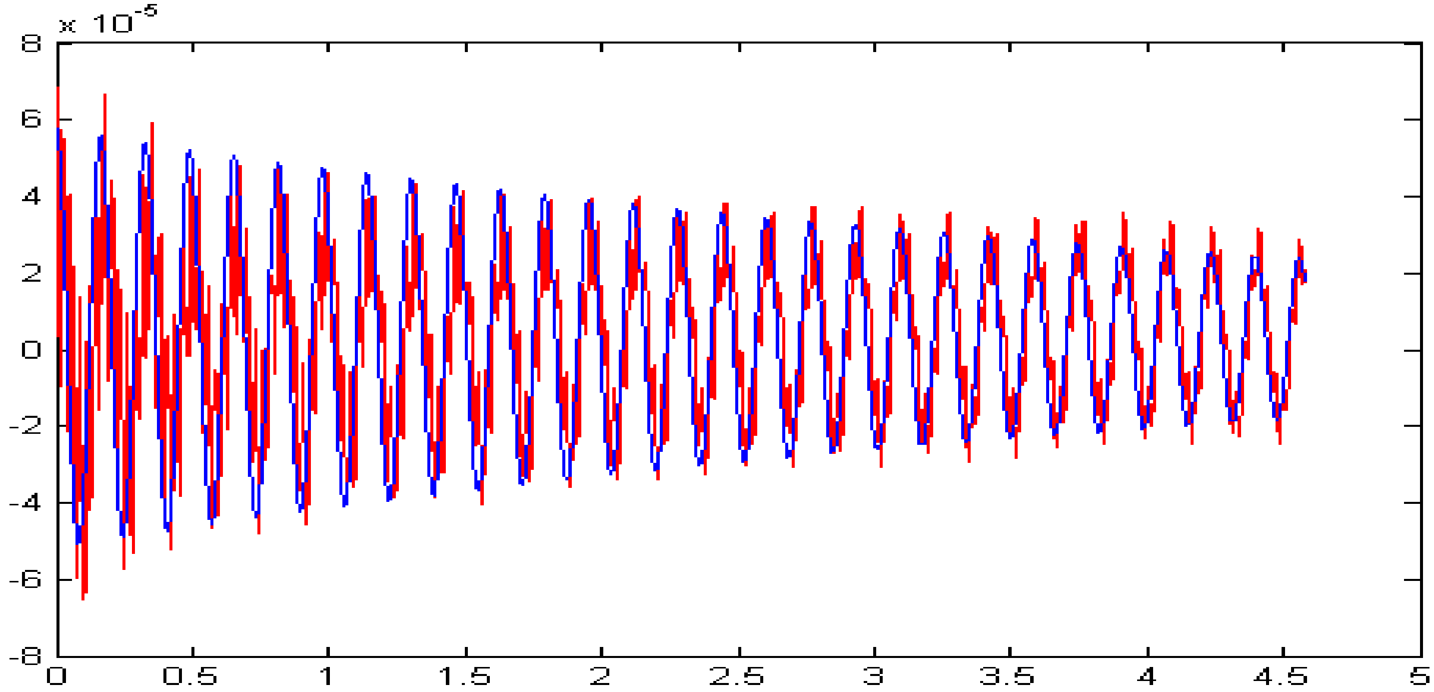
- The exponential regression gave us the expression .832*$e^{-.21t}$
- Our formula then became

$$\hat{y} = 0.832 * e^{-.21t} * \cos(2\pi * \frac{28}{4.558} t)$$

- Our model needed to be modified to "de-standardize" the data, giving

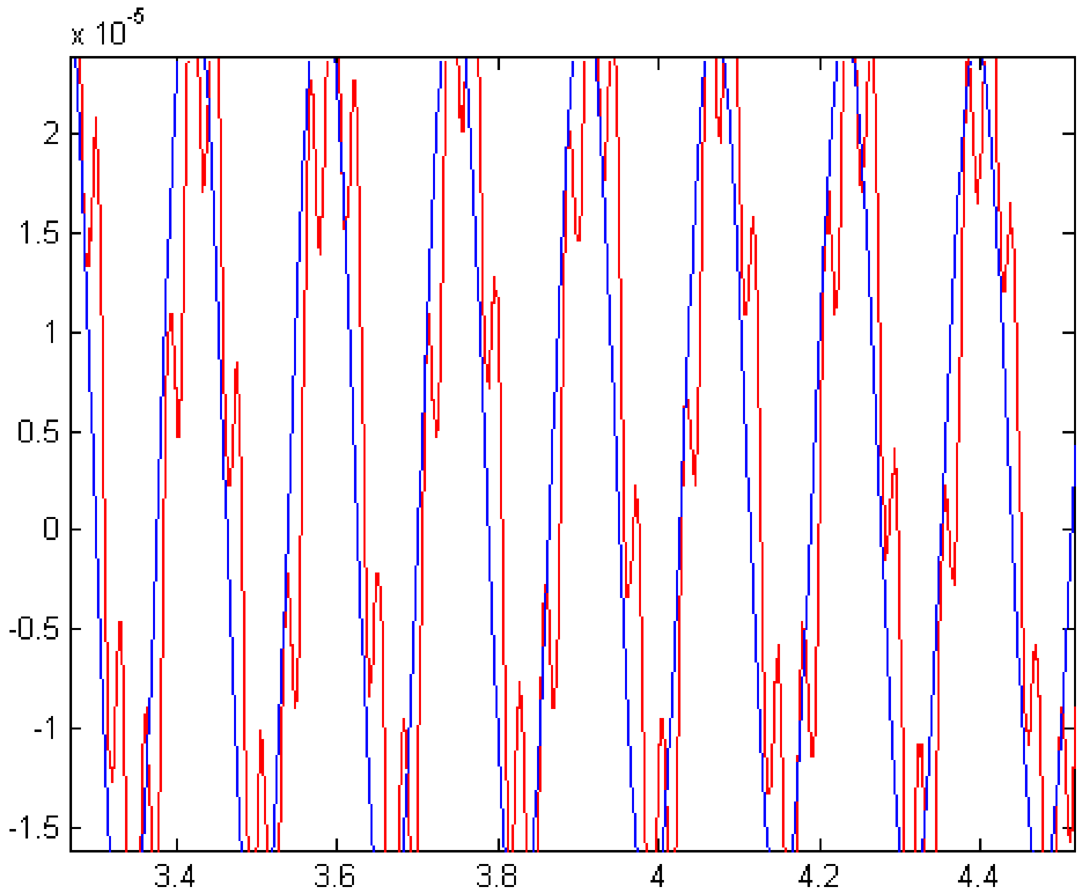
$$\hat{y} = 2.9242 * 10^{-6} + 6.5695 * 10^{-5} (0.832 * e^{-.21t} * \cos(2\pi * \frac{28}{4.558} t))$$

Oops

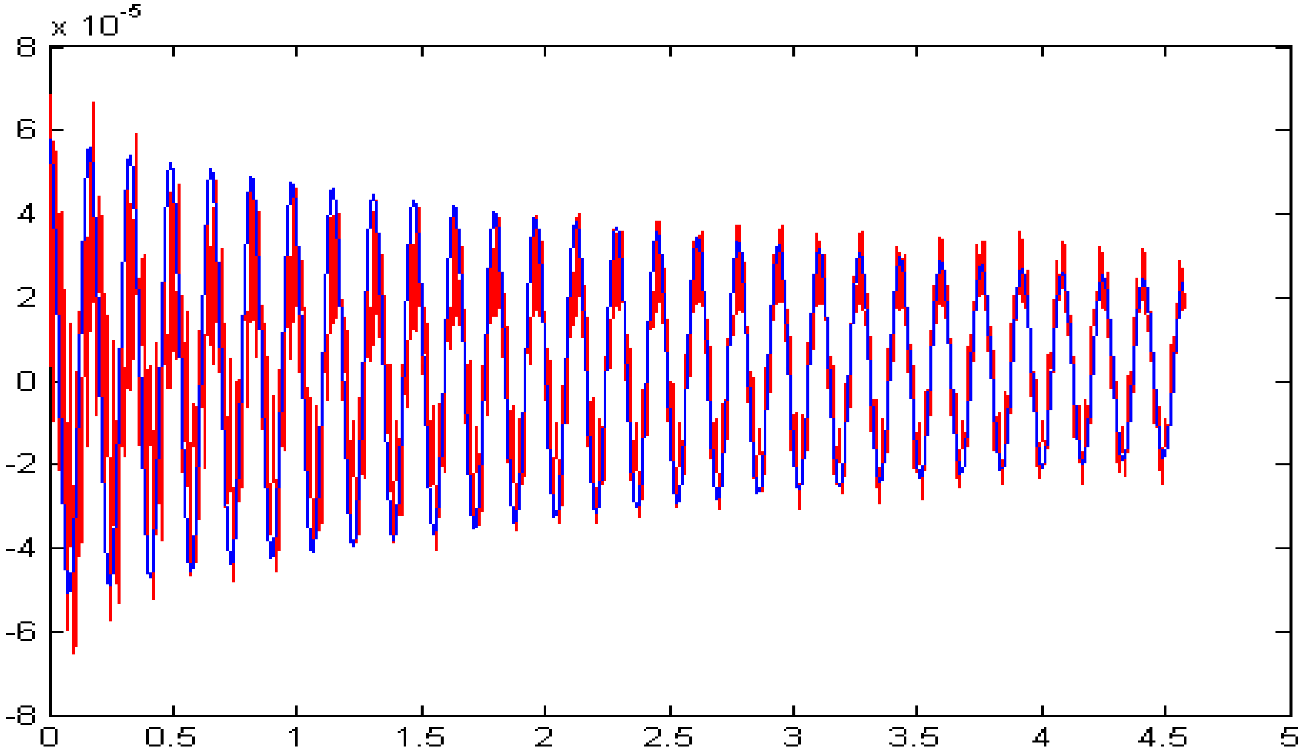


- Our model proved to be slightly off from the original data
- We instead took T = 4.412/27, from the next-to-last local max, which fit the data much better

Close-up

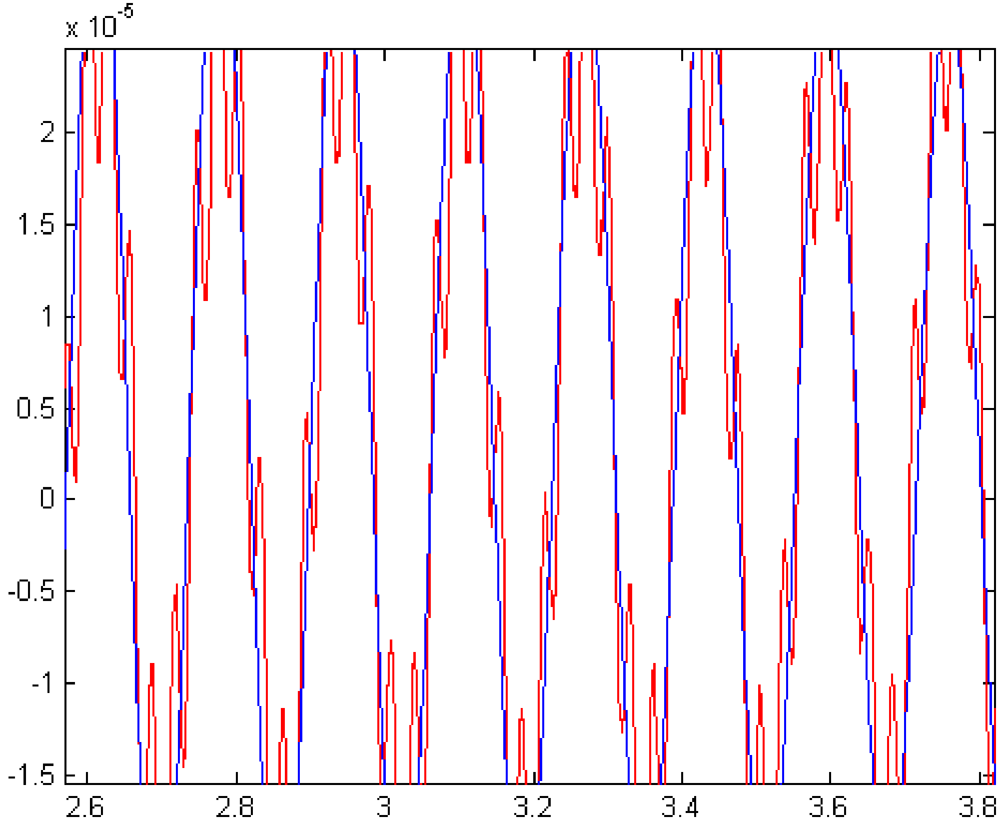


The final equation



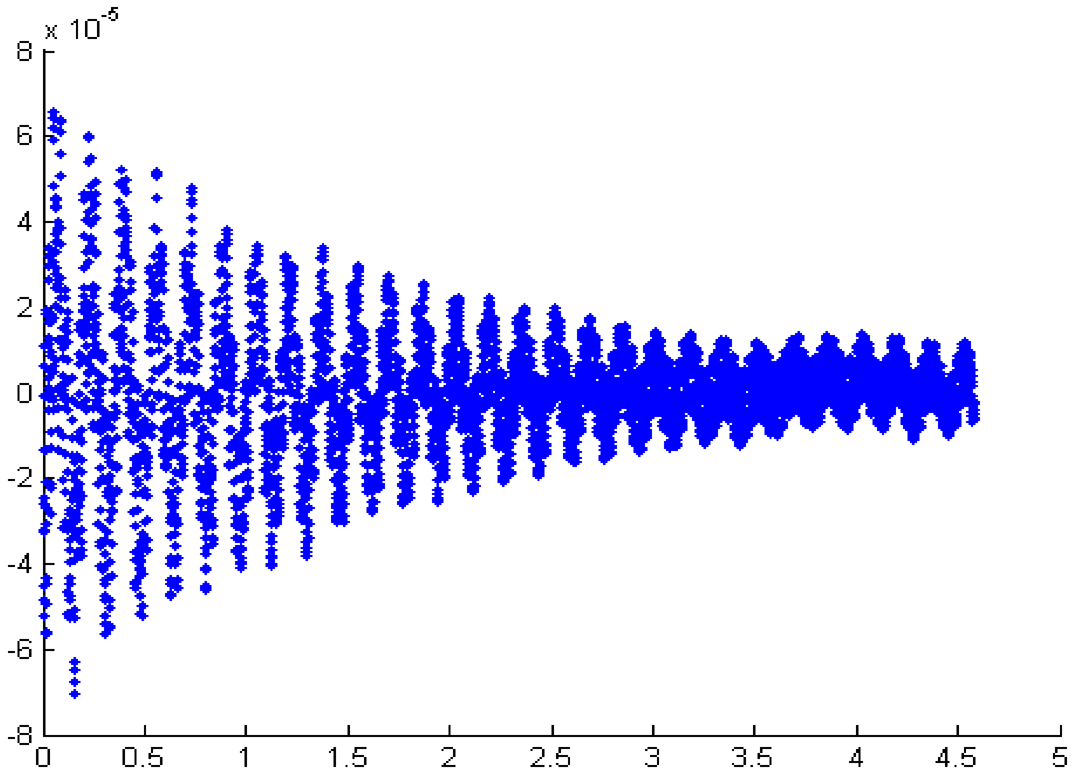
$$\hat{y} = 2.9242 * 10^{-6} + 6.5695 * 10^{-5} (0.832 * e^{-.21t} * \cos(2\pi * \frac{27}{4.412} t))$$

The final equation (close-up)



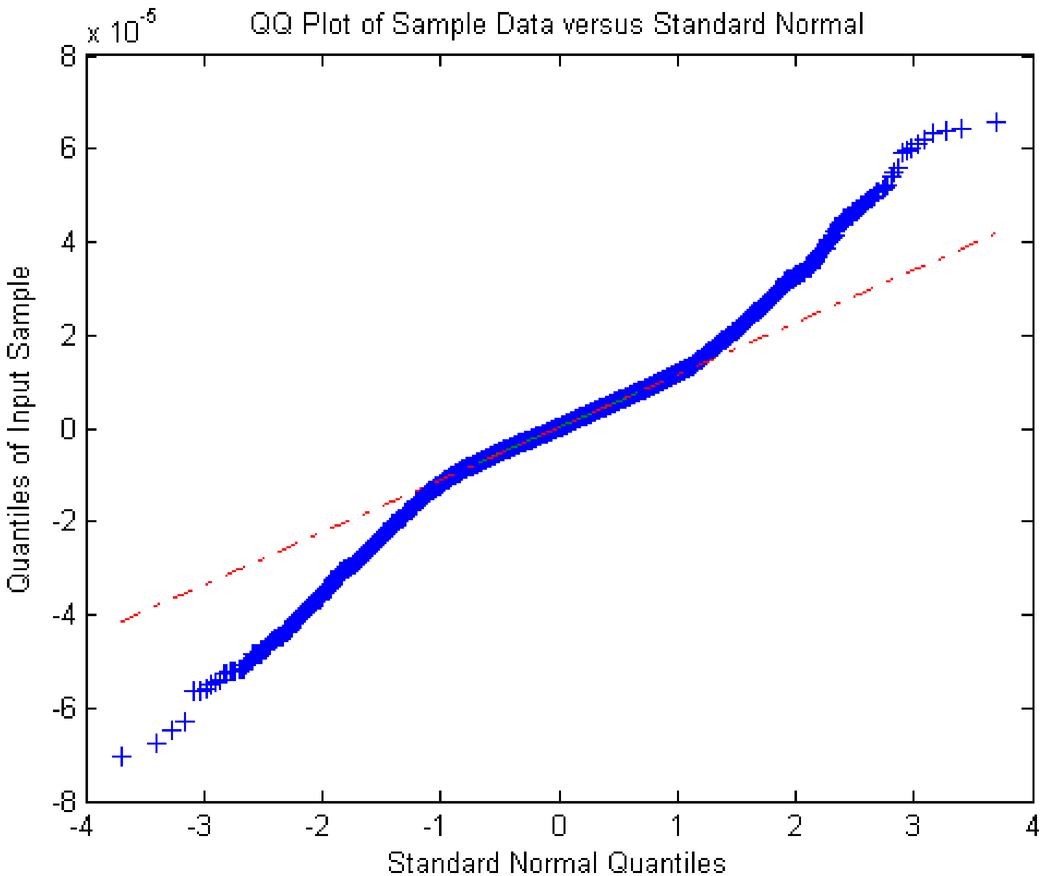
$$\hat{y} = 2.9242 * 10^{-6} + 6.5695 * 10^{-5} (0.832 * e^{-.21t} * \cos(2\pi * \frac{27}{4.412} t))$$

Residuals



- Scatter plot of $y - \hat{y}$ (residuals)

More residuals



- QQ-plot of residuals

Conclusions

- Our assumptions:
  - The form used would produce the best model
  - Our model would fit into the characteristic spring equation
- Although utilizing different methods, we managed to develop a model that accurately describes the oscillation of the beam
  - Curve appears to fit the data well
  - Residuals are comparable to other models
- Because of the method used, it would have been difficult to attempt to optimize our equation