Acoustic Attenuation Employing Variable Wall Admittance

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Abstract

In this discussion we present results for sound attenuation in a half plane using arrays of micro-acoustic actuators on a control surface. We explain and use a computational method developed in a previous paper [2]. Specifically, we carry out computations that document the sensitivity of the overall energy dissipation as a function of wall admittance.

1 Introduction

A recent topic of much interest focuses on the use of arrays of small acoustic sources, which could be constructed of fluidic devices [1] or piezoelectric materials, for sound absorption. In such an array, each element would be locally reacting by feedback of a pressure measurement via an appropriate compensator to the elemental acoustic source.

In this paper we report on the use of a computational technique developed in [2] that can be employed to analyze the absorption rate of acoustic arrays. We present computational findings that suggest enhanced energy dissipation via locally varying admittances on the boundary. These results are a continuous effort in the development of a conceptual framework for design, analysis, and implementation of *smart* or *adaptive* acoustic arrays to be used in noise control in structural systems.

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2 Problem Formulation

In this paper we consider a typical physical scenario for sound absorption or reflection on a treated planar wall. The sound pressure p satisfies the acoustic wave equation

$$p_{\tau\tau} - c^2 \nabla^2 p = 0, \tag{2.1}$$

and pressure and velocity are related through Euler's equation

$$\rho \mathbf{v}_\tau = -\nabla p, \tag{2.2}$$

where c is the speed of sound in air and ρ is the mass density of air. The vector $\mathbf{v} = (u, v, w)$ is the particle velocity of air, τ denotes unscaled time and $\mathbf{v}_\tau = \partial \mathbf{v}/\partial\tau$; see [3], [5], and [6].

We consider this system in an infinite half plane bounded by an infinite rigid wall located at $x = 0$ treated with an acoustic array that is finite and symmetric about the origin, $y = 0$. We assume that the acoustic array is sufficiently long in the z-direction, making the system essentially independent of z and thus two dimensional, as depicted in Figure 1. We define the domain $\Omega = \{(x, y) : x \geq 0, y \in \Re\}$ for our problem.

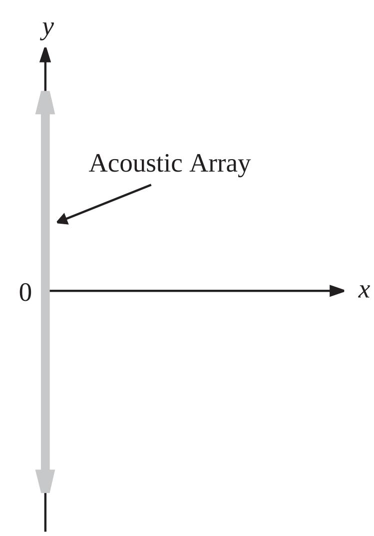

Figure 1: An acoustic array in half plane acoustic domain.

The interaction of waves with the boundary is described by the boundary condition

$$\mathbf{v}(0, y, \tau) \cdot \vec{\imath} = u(0, y, \tau) = -g(y)p(0, y, \tau), \qquad -\infty < y < \infty, \tau > 0. \tag{2.3}$$

This boundary condition follows from the *normal input admittance*, which is defined as the ratio of the particle velocity normal to the wall to the pressure [4], where g is the interaction admittance. We observe that $g \equiv 0$ in the case of a rigid boundary, while $g \to \infty$ corresponds to a pressure release boundary, $p \equiv 0$.

Let ϕ be the velocity potential, $\mathbf{v} = \nabla\phi$. Then from (2.2) we have $p = -\rho\phi_\tau$ and one can readily argue that ϕ satisfies

$$\phi_{\tau\tau} - c^2\nabla^2\phi = 0 \qquad \text{in } \Omega, \tag{2.4}$$

$$\phi_x(0,y,\tau) = \rho\, g(y)\, \phi_\tau(0,y,\tau), \qquad -\infty < y < \infty, \tau > 0. \tag{2.5}$$

We scale time by the sound speed, letting $t = c\tau$, and we define $\gamma(y) = \rho c\, g(y)$, so that (2.4) and (2.5) become

$$\phi_{tt} - \nabla^2\phi = 0 \qquad \text{in } \Omega \tag{2.6}$$

$$\phi_x(0,y,t) = \gamma(y)\, \phi_t(0,y,t), \qquad -\infty < y < \infty, t > 0. \tag{2.7}$$

The magnitude of the admittance in a given direction is a function of the wave propagation angle. Let $\mathbf{x}$ and $\mathbf{k}$ denote the position vector (x, y) and the wavenumber vector (k, l), respectively. Then for a plane wave, denoted $\phi(\mathbf{x}, t) = \phi_o\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})}$, the normalized wave admittance in the x-direction is

$$\beta = \rho c \frac{u}{p} = \frac{k}{\omega} = \frac{k}{|\mathbf{k}|}, \tag{2.8}$$

since $\omega = |\mathbf{k}|$. We note that $0 \le |\beta| \le 1$ with $|\beta| = 0$ corresponding to a plane wave traveling only in the y-direction and $|\beta| = 1$ corresponding to a plane wave traveling only in the x-direction. In order to absorb sound at the boundary we might choose the normal input admittance γ to match the normal wave admittance β. However, in general, the knowledge of the angle of incidence of the incoming wave is unknown. Moreover, for most fields the angle of incidence is arbitrary. For this reason, in our model we test the absorption characteristics for a given γ by the absorption rate of an ideal reverberant sound field under the assumption that the angle of incidence is uniformly distributed among all directions.

3 Frequency Domain and Approximation

To facilitate approximations and numerical computations, we reformulate equations (2.6) and (2.7) in the frequency domain. Since the spatial domain Ω is the half plane $x > 0$, then without loss of generality, we use the Fourier cosine transform with respect to x and full Fourier transform with respect to y. Let $\hat{\phi}$ be the Fourier cosine/full transform of ϕ,

$$\hat{\phi}(\mathbf{k}, t) = \hat{\phi}(k, l, t) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \phi(x, y, t) \cos(kx)\, e^{-ily}\, dx\, dy. \tag{3.1}$$

Then the inverse transform is given by

$$\phi(\mathbf{x}, t) = \phi(x, y, t) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{0}^{\infty} \hat{\phi}(k, l, t) \cos(kx)\, e^{ily}\, dk\, dl. \tag{3.2}$$

We can readily derive an equation for $\hat{\phi}(k, l, t)$. As detailed in [2], we obtain

$$\hat{\phi}_{tt}(k, l, t) = -(k^2 + l^2)\hat{\phi}(k, l, t) - \int_{-\infty}^{\infty} \int_{0}^{\infty} \hat{\phi}_t(\alpha, \beta, t)\, \hat{\gamma}(l, \beta)\, d\alpha\, d\beta, \tag{3.3}$$

where

$$\hat{\gamma}(l, \beta) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \gamma(y) e^{-i(l-\beta)y}\, dy.$$

Since the acoustic array is assumed to be symmetric, $\gamma(y)$ is an even function and thus

$$\hat{\gamma}(l, \beta) = \frac{2}{\pi^2} \int_{0}^{\infty} \gamma(y) \cos[(l - \beta)y]\, dy = \tag{3.4}$$
$$\frac{2}{\pi^2} \int_{0}^{\infty} \gamma(y)[\cos(ly)\cos(\beta y) + \sin(ly)\sin(\beta y)]\, dy.$$

If we further assume that the initial field is an even function of y, then the map $y \to \phi(\cdot, y, \cdot)$ is even, and we only need to work with the nonnegative values of l. Moreover, the integrand $\gamma(y)\sin(ly)\sin(\beta y)$ in (3.4) is an odd function of β and thus does not contribute to the integral term of (3.3) since $\beta \to \hat{\phi}_t(\alpha, \beta, t)$ is even. As a result, equation (3.3) reduces to

$$\hat{\phi}_{tt}(k, l, t) = -(k^2 + l^2)\hat{\phi}(k, l, t) - \int_{0}^{\infty} \int_{0}^{\infty} \hat{\phi}_t(\alpha, \beta, t)\, \Gamma(l, \beta)\, d\alpha\, d\beta, \tag{3.5}$$

where

$$\Gamma(l, \beta) = \frac{4}{\pi^2} \int_{0}^{\infty} \gamma(y) \cos(ly) \cos(\beta y)\, dy. \tag{3.6}$$

We note that this yields a nonstandard integro-partial differential equation for $\hat{\phi}$ for which approximation techniques must be developed. We summarize a semi-discrete finite element approximation based on piecewise constant elements (zero-order splines) that was developed and tested numerically in [2].

For finite positive integers K and L, we consider the truncated finite domain in frequency space

$$\Omega_{k,l} = \{(k,l) : 0 \le k \le K, 0 \le l \le L\}. \tag{3.7}$$

We discretize (3.5) in this finite domain.

Let $0 = k_0 < k_1 < k_2 < \cdots < k_M = K$ and $0 = l_0 < l_1 < l_2 < \cdots < l_N = L$ be partitions along the k- and l-axis in the kl-plane. Let $\Delta k_i = k_i - k_{i-1}$, $\Delta l_j = l_j - l_{j-1}$, and $R_{ij} = (k_{i-1}, k_i] \times (l_{j-1}, l_j]$. Let $(\tilde{k}_i, \tilde{l}_j)$ denote the midpoint of each cell R_{ij}.

In equation (3.5), we apply the (piecewise constant in frequency) approximation

$$\hat{\phi}(k,l,t) \approx \Phi^{MN}(k,l,t) = \sum_{i=1}^{M} \sum_{j=1}^{N} \Phi_{ij}^{MN}(t) \chi_{ij}^{MN}(k,l). \tag{3.8}$$

where

$$\chi_{ij}^{MN}(k,l) = \begin{cases} 1 & (k,l) \in R_{ij}, \\ 0 & \text{otherwise,} \end{cases}$$

and $\Phi_{ij}^{MN}(t)$ is an approximation of $\hat{\phi}(\tilde{k}_i, \tilde{l}_j, t)$. Then, we evaluate (3.8) at the midpoint of each cell R_{ij}. This results a the system of equations of the form

$$\ddot{\Phi}_{ij}^{MN}(t) + \lambda_{ij}^2 \Phi_{ij}^{MN}(t) + \sum_{n=1}^{N} \left(g_{jn}^{MN} \sum_{m=1}^{M} \Delta k_m \dot{\Phi}_{mn}^{MN}(t) \right) = 0, \tag{3.9}$$

where

$$\lambda_{ij} = \sqrt{\tilde{k}_i^2 + \tilde{l}_j^2}, \qquad g_{jn}^{MN} = \int_{l_{n-1}}^{l_n} \Gamma(\tilde{l}_j, \beta) d\beta.$$

We remark that we use the superscript MN with the variables in (3.9) since they depend on the discretization in (k,l). For notational convenience, we shall suppress this superscript in the remainder of our discussions, whenever no confusion will result.

We can write the system (3.9) in matrix form by introducing the $\Re^{MN}$ vector

$$\Phi = [\Phi_{11}, \Phi_{21}, \dots, \Phi_{M1}, \Phi_{12}, \Phi_{22}, \dots, \Phi_{M2}, \dots, \Phi_{1N}, \Phi_{2N}, \dots, \Phi_{MN}]^T,$$

the $MN \times MN$ diagonal matrix

$$\Lambda = \mathrm{diag}(\lambda_{11}^2, \lambda_{21}^2, \ldots, \lambda_{M1}^2, \lambda_{12}^2, \lambda_{22}^2, \ldots, \lambda_{M2}^2, \ldots, \lambda_{1N}^2, \lambda_{2N}^2, \ldots, \lambda_{MN}^2),$$

and the damping matrix

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1N} \\ g_{21} & g_{22} & \cdots & g_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ g_{N1} & g_{N2} & \cdots & g_{NN} \end{bmatrix}.$$

We define the $M \times M$ matrix

$$\Delta_k = \begin{bmatrix} \Delta k_1 & \Delta k_2 & \cdots & \Delta k_M \\ \Delta k_1 & \Delta k_2 & \cdots & \Delta k_M \\ \vdots & \vdots & \cdots & \vdots \\ \Delta k_1 & \Delta k_2 & \cdots & \Delta k_M \end{bmatrix},$$

and let D be the Kronecker tensor product

$$D = \mathrm{kron}(G, \Delta_k).$$

Then, the second order system (3.9) can be written in the matrix form

$$\ddot{\Phi}(t) + D\dot{\Phi}(t) + \Lambda\Phi(t) = 0, \tag{3.10}$$

or, as a first order system by defining

$$Z(t) = \begin{bmatrix} \Phi(t) \\ \hline \dot{\Phi}(t) \end{bmatrix}$$

to obtain the equation

$$\dot{Z} = AZ. \tag{3.11}$$

Here, A is the $(2MN) \times (2MN)$ matrix defined by

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -\Lambda & -D \end{array}\right],$$

and I is the $(MN) \times (MN)$ identity matrix. As in [2], we use this approximate system in the calculations described here.

In all of our investigations, we assume that the initial sound field is random and the spatial autocorrelation is independent of the position and phase, being a function only of the distance between two points in space. Thus, the initial random field $\phi(\mathbf{x}, 0)$ satisfies

$$\mathcal{E}\left[\phi(\mathbf{x}, 0)\, \phi(\mathbf{x}+\mathbf{r}, 0)\right] = R(|\mathbf{r}|)$$

for all $\mathbf{x}$, where $\mathcal{E}[\cdot]$ denotes the expectation. In the frequency domain this is equivalent to

$$\mathcal{E}\left[\overline{\hat{\phi}(\mathbf{k}, 0)}\, \hat{\phi}(\mathbf{k}', 0)\right] = (2\pi)^2\, \delta(\mathbf{k}-\mathbf{k}') \int R(|\mathbf{r}|)\, e^{-i\mathbf{k}\cdot\mathbf{r}}\, d\mathbf{r}, \tag{3.12}$$

where δ is the Dirac delta function.

From this it follows that a natural condition to require on the approximating random Fourier components $\{Z_i(0)\}$ for the system (3.11) is

$$\mathcal{E}\left[Z_i(0)\, Z_j(0)\right] = 0, \quad \text{if } i \neq j. \tag{3.13}$$

If we then define the corresponding approximate correlation matrix

$$C(t) = \mathcal{E}\left[Z(t)\, Z^T(t)\right], \tag{3.14}$$

we see that (3.13) implies that $C(0)$ is diagonal. Thus, we can write

$$C(0) = \sum_{\nu=1}^{2MN} w_\nu\, e^{(\nu)} \left(e^{(\nu)}\right)^T, \tag{3.15}$$

where $e^{(\nu)}$ is the unit vector in $\Re^{2MN}$ and $W = (w_\nu)$ is the vector of the initial field discrete power spectral densities.

We then see that the $2MN \times 2MN$ matrix function $C(t)$ satisfies the matrix Lyapunov differential equation system

$$\dot{C}(t) = A\, C(t) + C(t)\, A^T, \tag{3.16}$$

which is a large system of coupled equations with dimension $(2MN)^2$.

The solution of (3.16) can be obtained in superposition form (see [2] for details)

$$C(t) = \sum_{\nu=1}^{2MN} w_\nu\, F^{(\nu)}(t) \left(F^{(\nu)}(t)\right)^T, \tag{3.17}$$

where the vector $F^{(\nu)}(t)$ for each ν satisfies

$$\dot{F}^{(\nu)}(t) = A\,F^{(\nu)}(t), \qquad F^{(\nu)}(0) = e^{(\nu)}. \tag{3.18}$$

When considering fields with compact support Fourier components (such as those with finite bandwidth), as done here, the superposition formula (3.17) coupled with (3.18) is especially efficient.

4 Instantaneous Total Energy and Damping

Since our primary interest is to study attenuation provided by the acoustic array as we change the local admittance (as manifested in the shape of γ in (2.7)) it is most useful to have a measure of the total energy in the field.

We can define an approximation to the total energy for (2.6)-(2.7) by employing the total energy of system (3.10)

$$E^{MN}(t) = E_V^{MN}(t) + E_P^{MN}(t), \tag{4.1}$$

where the "kinetic" energy and "potential" energy are given, respectively, by

$$E_V^{MN}(t) = \frac{1}{2}\int |\nabla\phi^{MN}(\mathbf{x},t)|^2\,d\mathbf{x}, \qquad E_P^{MN}(t) = \frac{1}{2}\int [\phi_t^{MN}(\mathbf{x},t)]^2\,d\mathbf{x},$$

and $\phi^{MN}(\mathbf{x},t)$ is the inverse Fourier transform of $\Phi^{MN}(\mathbf{k},t)$ defined in (3.8). By Parseval's theorem, we have

$$E_V(t) = \frac{1}{2}\Phi^T(t)\,\Lambda\,\Phi(t), \qquad E_P(t) = \frac{1}{2}\dot{\Phi}^T(t)\,\dot{\Phi}(t), \tag{4.2}$$

where again we shall suppress the MN superscripts.

Now, by multiplying (3.10) by $\dot{\Phi}$, defining $E(t) = E_V(t) + E_P(t)$, and using (4.2), we can argue that

$$\dot{E}(t) = -\dot{\Phi}^T(t)\,D\,\dot{\Phi}(t) \le 0. \tag{4.3}$$

It is immediately apparent from (4.3) that the system is dissipative for any positive γ.

One can further argue (see [2] for details) that the expected energy can be approximated by

$$\mathcal{E}\,[E(t)] = \frac{1}{2}\,\mathcal{E}\left[\Phi^T(t)\,\Lambda\,\Phi(t) + \dot{\Phi}^T(t)\,\dot{\Phi}(t)\right] = \frac{1}{2}\,\mathrm{trace}[\mathcal{L}\,C(t)\,\mathcal{L}]. \tag{4.4}$$

where the matrix $\mathcal{L}$ is defined by

$$\mathcal{L} = \left[\begin{array}{c|c} \sqrt{\Lambda} & 0 \\ \hline 0 & I \end{array}\right].$$

Thus for a given γ, we can integrate (3.18) to obtain $F^{(\nu)}(t)$ and use (3.15), (3.17), and (4.4) to determine the decay rate of a field composed of the group of modes determined by a given bandwidth.

5 Computational Results

We turn finally to computations for a family of acoustic arrays using the methodology outlined in the previous sections. We consider a family of periodic acoustic arrays of length $2L_a$ in the y-direction and symmetric about the origin as depicted in Figure 2. The arrays consist of elements each of width w and maximum height h.

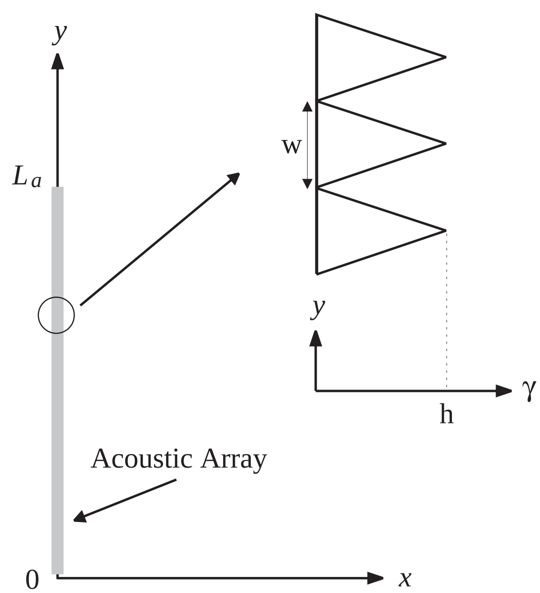

Figure 2: Acoustic array and the admittance function $\gamma(y) = \gamma_{(\mathrm{w},\mathrm{h})}(y)$.

Due to symmetry, we only need to consider the upper half ($y \geq 0$) of the arrays. The number of elements in a given half array is

$$N_e = L_a/\mathrm{w}.$$

For $n = 1, \ldots, N_e$, each nth element occupies the interval $\mathcal{I}_n = [(n-1)\mathrm{w}, n\mathrm{w}]$.

In each element we take the function $\gamma(y)$ to be a symmetric triangle curve of height h as shown in Figure 2. Thus we have

$$\gamma(y) = \begin{cases} \gamma^n(y), & y \in \mathcal{I}_n, \\ 0 & y > L_a, \end{cases} \tag{5.1}$$

where

$$\gamma^n(y) = \begin{cases} \frac{2\mathrm{h}}{\mathrm{w}} y - 2(n-1)\mathrm{h}, & y \in [(n-1)\mathrm{w}, (n-.5)\mathrm{w}], \\ -\frac{2\mathrm{h}}{\mathrm{w}} y + 2n\mathrm{h}, & y \in [(n-.5)\mathrm{w}, n\mathrm{w}]. \end{cases} \tag{5.2}$$

For our example calculations we used $L_a = 4$ meters along with various admittance widths w and heights h. Thus the family of admittance functions $\gamma = \gamma_{(\mathrm{w},\mathrm{h})}$ are parameterized by (w, h) as in (5.2).

Moreover we chose $K = L = 10$ and uniform partitions $\Delta k_i = \Delta l_j = 1$. This implies that $M = N = 10$ and that A in (3.11) has dimension 200×200. We calculated the entries of the matrix G by using the midpoint approximations

$$g_{jn} \approx \Delta l \, \Gamma(\tilde{l}_j, \tilde{l}_n) \;= \frac{4}{\pi^2} \, \Delta l \int_0^{L_a} \gamma(y) \cos(\tilde{l}_j y) \cos(\tilde{l}_n y) \, dy, \tag{5.3}$$

and then a quadrature Trapezoidal rule.

For our examples, we considered the bandwidth $3.5 \leq |\mathbf{k}| \leq 4.5$. The discrete modes that belong to this bandwidth are shown in Figure 3 where the dashed lines represent curves of constant $|\mathbf{k}|$. The set of these modes is

$$\Upsilon = \{(\tilde{k}_i, \tilde{l}_j) = (\, i - .5, j - .5\,) : (i,j) = (4,1), (4,2), (3,3), (4,3), (1,4), (2,4), (3,4)\}.$$

We assume that the initial field consists only of the velocity field. Thus for the modes in Υ the corresponding set of $\nu = i + 10(j-1)$ values is

$$\mathcal{N} = \{4, 14, 23, 24, 31, 32, 33\},$$

and for the coefficients w_ν we have

$$w_\nu = 0, \qquad \nu \notin \mathcal{N},$$

while w_ν, $\nu \in \mathcal{N}$, are given magnitudes that describe the initial field. For simplicity we considered a uniform initial energy distribution of magnitude one for each mode $\nu \in \mathcal{N}$. This leads to the instantaneous correlation

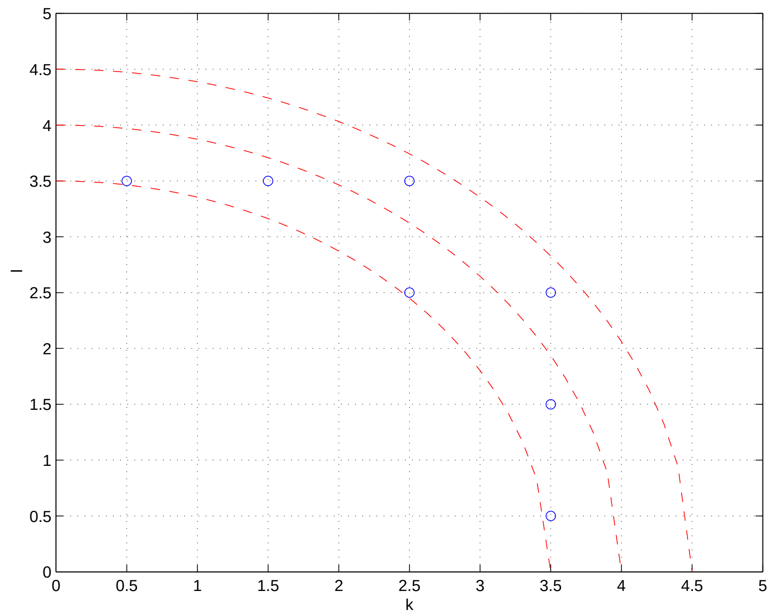

Figure 3: The set Υ of the initial discrete modes.

matrix given by (see (3.17) and (3.18))

$$C^{\mathcal{N}}(t) = \sum_{\nu \in \mathcal{N}} w_\nu F^{(\nu)}(t) \left(F^{(\nu)}(t) \right)^T ,$$

and the corresponding expected instantaneous total energy $E^{\mathcal{N}}$ given by

$$\mathcal{E}\left[E^{\mathcal{N}}(t)\right] = \frac{1}{2}\,\mathrm{trace}[\mathcal{L}\,C^{\mathcal{N}}(t)\mathcal{L}]. \tag{5.4}$$

Following the procedures given above and in [2], we computed the energy dissipation for a given admittance $\gamma_{(\mathrm{w},\mathrm{h})}$. We did this for a range of values of width $\mathrm{w} \in \mathcal{W} = \{.01, .02, .05, .1, .2, .5, 1, 1.333, 2, 4\}$ with fixed values of h and also for a range of values of height $\mathrm{h} \in \mathcal{H} = \{.1, .2, .5, 1, 2, 5, 10\}$ with fixed values of w, and plotted the normalized energy dissipation E_{20} at scaled time $t = 20$ given by

$$E_{20} = 10 \log \left[\frac{\mathcal{E}\left[E^{\mathcal{N}}(t)\right]}{\mathcal{E}\left[E^{\mathcal{N}}(0)\right]} \right] \Bigg|_{t=20} .$$

Examples of our findings are plotted in Figure 4, where we depict dissipation for fixed height $\mathrm{h} = 2$ and various widths $\mathrm{w} \in \mathcal{W}$, and in Figure 5, where width was fixed at $\mathrm{w} = .01$ and $\mathrm{h} \in \mathcal{H}$.

These results are quite typical of our findings in which the energy dissipation exhibited much more sensitivity to changes in height than changes in width. A plot of the energy dissipation vs. (w, h) summarizes these findings in Figure 6.

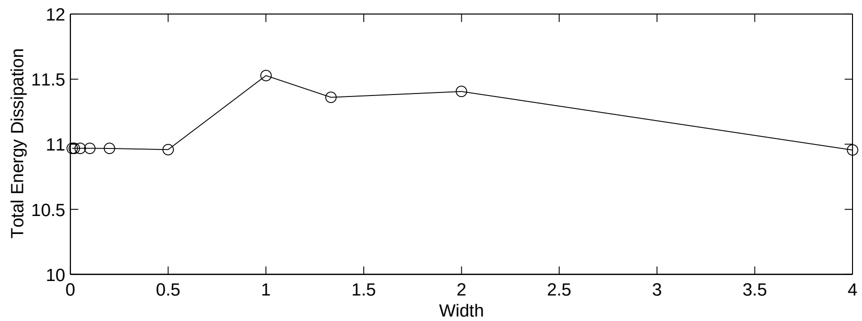

Figure 4: Energy dissipation for fixed height 2 and various widths $\in \mathcal{W}$

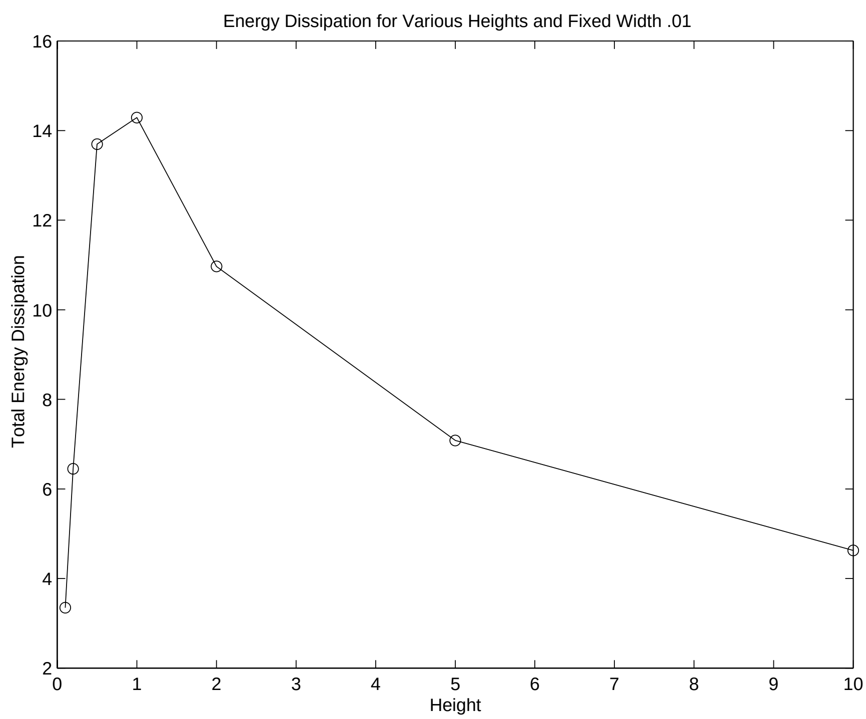

Figure 5: Energy dissipation for fixed width .01 and various heights $\in \mathcal{H}$

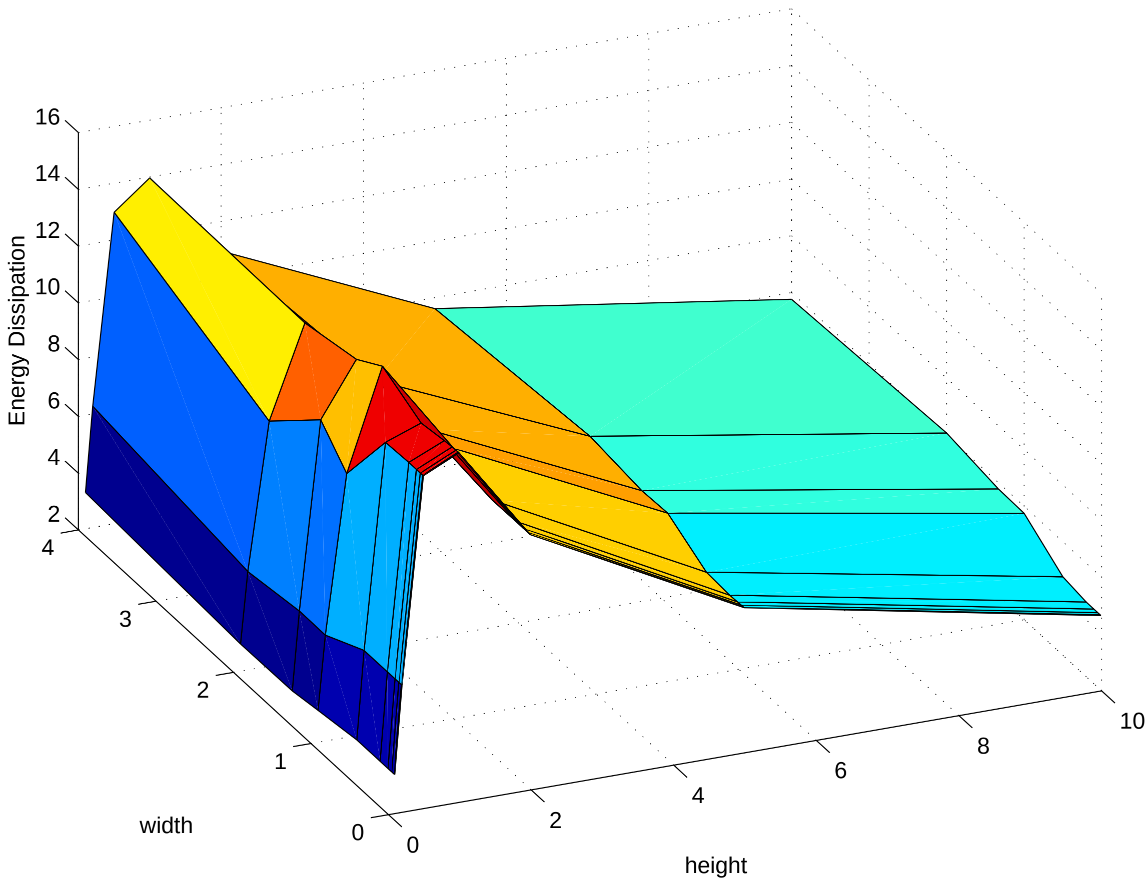

Figure 6: 3-D plot of energy dissipation with various widths and heights.

6 Concluding Remarks

The above results offer promise of the effective use of an array of micro-acoustic actuators as an absorbing surface for enclosed acoustic cavities.

We believe the computational findings discussed here provide a strong motivation for further investigations in the context of *smart* or *active control* in which the design parameter γ in (2.7) is allowed to vary in time and one seeks optimal strategies.

Acknowledgments

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