

**SOLVING THE VIBRATING BEAM  
INVERSE PROBLEM**

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# FORMING AN INVERSE PROBLEM

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## Mathematical Model:

$$\begin{aligned} \ddot{y} + C\dot{y} + Ky &= 0 \\ y(0) = y_0, \quad \dot{y}_0 &= v_0 \end{aligned} \tag{1}$$

## Experimental Data:

- displacement of the vibrating beam
- similar to  $y$  in the model
- denote collected data as  $y_i$  and  $y(t_i)$  in the model as  $y(t_i; C, K)$

## Cost Function:

$$J(C, K) = \frac{1}{2} \sum_{i=1}^N (y(t_i; C, K) - y_i)^2$$

## Optimization Problem:

$$\text{minimize } J(C, K) \Rightarrow \min \frac{1}{2} \sum_{i=1}^N (y(t_i; C, K) - y_i)^2$$

# SOLVING AN INVERSE PROBLEM

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- Brute Force

- Construct  $\dot{y}_t \approx \frac{y_t - y_{t-1}}{\Delta t}$

- Construct  $\ddot{y}_t \approx \frac{\dot{y}_t - \dot{y}_{t-1}}{\Delta t}$

- Minimize

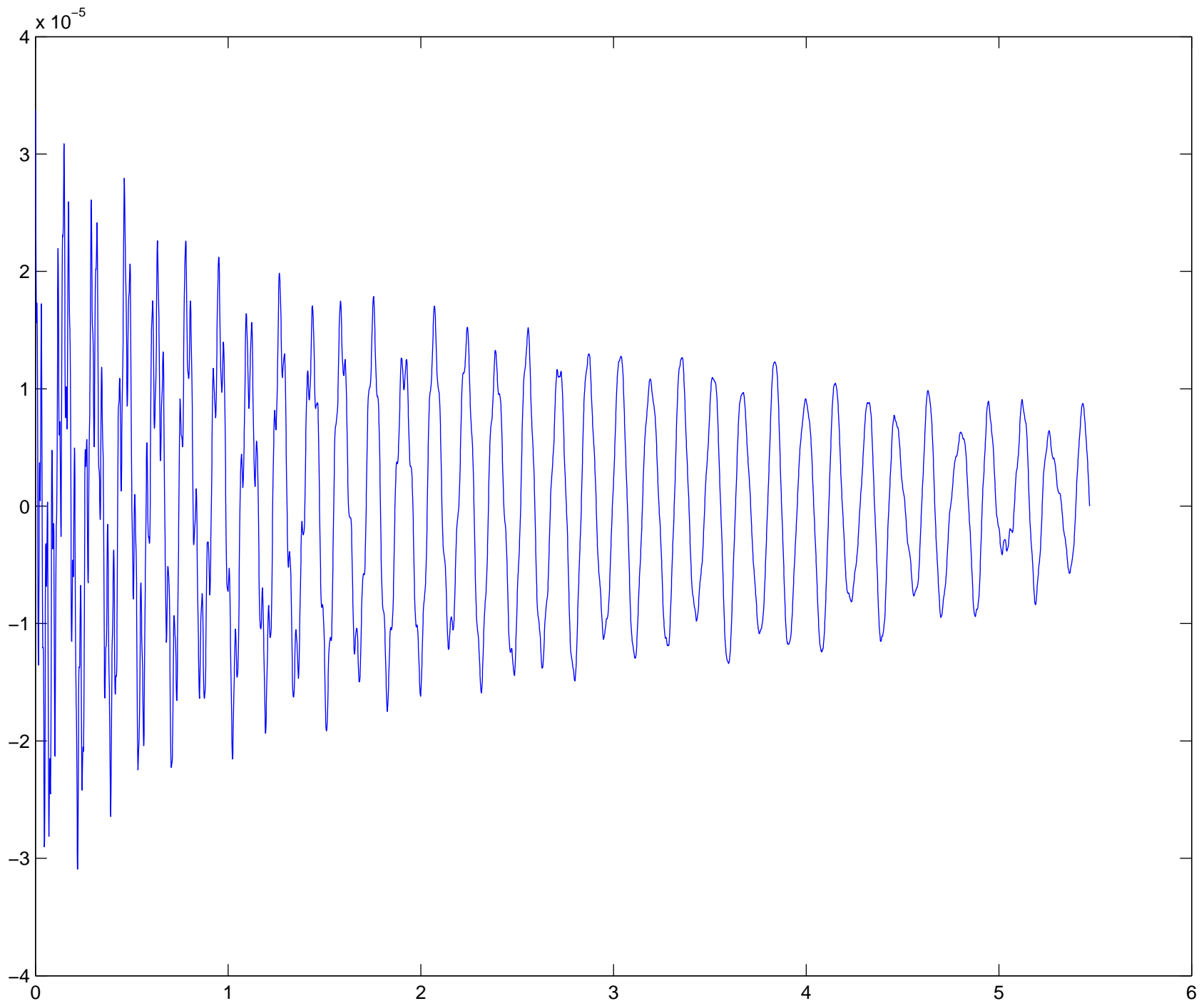
$$\sum_{i=1}^N (\ddot{y}_i - K\dot{y}_i - Cy_i)^2$$

**What's wrong with this approach?**

# SOLVING AN INVERSE PROBLEM

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- **Measure the same things**
  - Analyze experimental data
  - Adjust either the data or the simulations
- **Solve the differential equations from the model**
- **Choose an optimization method**
- **Make an educated initial guess on the parameters (C,K)**



# SOLVING THE MATHEMATICAL MODEL

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- **Mathematical Model:**

$$\ddot{y} + C\dot{y} + Ky = 0 \tag{2}$$

$$y(0) = y_0, \quad \dot{y}_0 = v_0$$

- **Constructing a System of 1<sup>st</sup> ODE:**

Let  $z_1 = y$ ,  $z_2 = \dot{y}$ , and  $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$$\frac{d}{dt} \begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K & -C \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = \begin{bmatrix} y_0 \\ v_0 \end{bmatrix} \tag{3}$$

- Use *ode23*:  $[t,z]=ode23(@ode\_model,tspan,z_0,[ ],C,K)$

tspan =

$z_0 =$

# INVERSE PROBLEMS AND INITIAL GUESSES

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- Cost Function:  $\text{cost} = \sum_{i=1}^N (y(t_i, q) - y_i)^2$

- Minimize Cost Function:  $q = [C, K]$

$$[q, \text{cost}] = \text{fminsearch}(@\text{cost\_beam}, q_0, [], \text{time}, y)$$

- How do we choose  $q_0$ ?

$$\text{Analytic Result: } y(t) = e^{-\frac{1}{2}Ct} A \sin(\omega t + \phi); \quad \omega = \frac{\sqrt{4K - C^2}}{2}$$

◇  $C$ - related to damping coefficient

◇  $K$ - related to spring constant

- What does a small  $C$  imply? What about a large  $C$ ?
- What does a small  $K$  imply? What about a large  $K$ ?

# DATA ANALYSIS

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- Data does not start at max displacement
- Why?
- How do we fix it?

Max Displacement: - find max value in the data and start data at the max value

```
maxval=max(data); % find max value in the data
```

```
index=find(data==maxval); %find where max value in the data is located
```

```
 $y_i$ =data(index:end); %reset the data so data starts at max value
```

```
time=time(index:end); %need to reset the time to correspond with data
```

Oscillating Around Zero

```
 $y_i = y_i - \text{mean}(y_i)$ ; %(shift data down)
```